

The Pythagorean Theorem

MODULE



12



ESSENTIAL QUESTION

How can you use the Pythagorean Theorem to solve real-world problems?

LESSON 12.1

The Pythagorean Theorem



FL 8.G.2.6, 8.G.2.7

LESSON 12.2

Converse of the Pythagorean Theorem



FL 8.G.2.6

LESSON 12.3

Distance Between Two Points



FL 8.G.2.8



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Real-World Video

The sizes of televisions are usually described by the length of the diagonal of the screen. To find this length of the diagonal of a rectangle, you can use the Pythagorean Theorem.

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Find the Square of a Number

EXAMPLE Find the square of 2.7. *Multiply the number by itself.*

$$\begin{array}{r} 2.7 \\ \times 2.7 \\ \hline 189 \\ 54 \\ \hline 7.29 \end{array}$$

So, $2.7^2 = 7.29$.

Find the square of each number.

1. 5 _____ 2. 16 _____ 3. -11 _____ 4. $\frac{2}{7}$ _____

Order of Operations

EXAMPLE $\sqrt{(5 - 2)^2 + (8 - 4)^2}$ *First, operate within parentheses.*

$$\sqrt{(3)^2 + (4)^2}$$

Next, simplify exponents.

$$\sqrt{9 + 16}$$

Then add and subtract left to right.

$$\sqrt{25}$$

Finally, take the square root.

$$5$$

Evaluate each expression.

5. $\sqrt{(6 + 2)^2 + (3 + 3)^2}$ _____ 6. $\sqrt{(9 - 4)^2 + (5 + 7)^2}$ _____

7. $\sqrt{(10 - 6)^2 + (15 - 12)^2}$ _____ 8. $\sqrt{(6 + 9)^2 + (10 - 2)^2}$ _____

Simplify Numerical Expressions

EXAMPLE $\frac{1}{2}(2.5)^2(4) = \frac{1}{2}(6.25)(4)$ *Simplify the exponent.*

$$= 12.5$$

Multiply from left to right.

Simplify each expression.

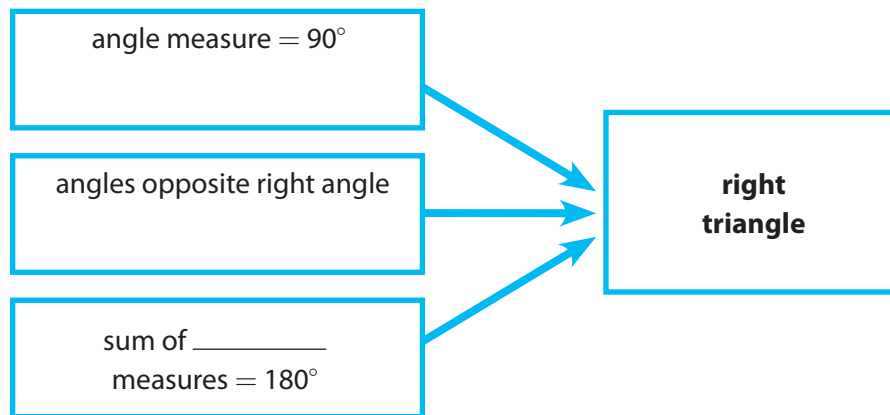
9. $5(8)(10)$ _____ 10. $\frac{1}{2}(6)(12)$ _____ 11. $\frac{1}{3}(3)(12)$ _____

12. $\frac{1}{2}(8)^2(4)$ _____ 13. $\frac{1}{4}(10)^2(15)$ _____ 14. $\frac{1}{3}(9)^2(6)$ _____

Reading Start-Up

Visualize Vocabulary

Use the ✓ words to complete the graphic.



Understand Vocabulary

Match the term on the left to the correct expression on the right.

- | | |
|---------------|---|
| 1. hypotenuse | A. An idea that has been demonstrated as true. |
| 2. theorem | B. The two sides that form the right angle of a right triangle. |
| 3. legs | C. The side opposite the right angle in a right triangle. |

Vocabulary

Review Words

- ✓ acute angles (*ángulos agudos*)
- ✓ angles (*ángulos*)
- area (*área*)
- ordered pair (*par ordenado*)
- ✓ right angle (*ángulo recto*)
- ✓ right triangle (*triángulo recto*)
- square root (*raíz cuadrada*)
- x-coordinate (*coordenada x*)
- y-coordinate (*coordenada y*)

Preview Words

- hypotenuse (*hipotenusa*)
- legs (*catetos*)
- theorem (*teorema*)
- vertex (*vértice*)

Active Reading

Booklet Before beginning the module, create a booklet to help you learn about the Pythagorean Theorem. Write the main idea of each lesson on each page of the booklet. As you study each lesson, write important details that support the main idea, such as vocabulary and formulas. Refer to your finished booklet as you work on assignments and study for tests.





Unpacking the Standards

Understanding the standards and the vocabulary terms in the standards will help you know exactly what you are expected to learn in this module.

FL 8.G.2.7

Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions.

Key Vocabulary**Pythagorean Theorem**

(Teorema de Pitágoras)

In a right triangle, the square of the length of the hypotenuse is equal to the sum of the squares of the lengths of the legs.

What It Means to You

You will find a missing length in a right triangle, or use side lengths to see whether a triangle is a right triangle.

UNPACKING EXAMPLE 8.G.2.7

Mark and Sarah start walking at the same point, but Mark walks 50 feet north while Sarah walks 75 feet east. How far apart are Mark and Sarah when they stop?

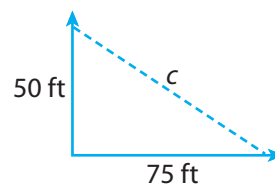
$$a^2 + b^2 = c^2$$

$$50^2 + 75^2 = c^2$$

$$2500 + 5625 = c^2$$

$$8125 = c^2$$

$$90.1 \approx c$$



Pythagorean Theorem

Substitute.

Mark and Sarah are approximately 90.1 feet apart.

FL 8.G.2.8

Apply the Pythagorean Theorem to find the distance between two points in a coordinate system.

Key Vocabulary**coordinate plane**

(plano cartesiano)

A plane formed by the intersection of a horizontal number line called the x-axis and a vertical number line called the y-axis.

What It Means to You

You can use the Pythagorean Theorem to find the distance between two points.

UNPACKING EXAMPLE 8.G.2.8

Find the distance between points A and B.

$$(AC)^2 + (BC)^2 = (AB)^2$$

$$(4 - 1)^2 + (6 - 2)^2 = (AB)^2$$

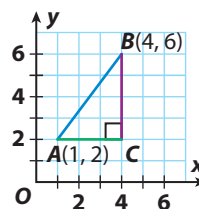
$$3^2 + 4^2 = (AB)^2$$

$$9 + 16 = (AB)^2$$

$$25 = (AB)^2$$

$$5 = AB$$

The distance is 5 units.



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LESSON 12.1 The Pythagorean Theorem

 **FL** 8.G.2.7

Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions. Also 8.G.2.6



ESSENTIAL QUESTION

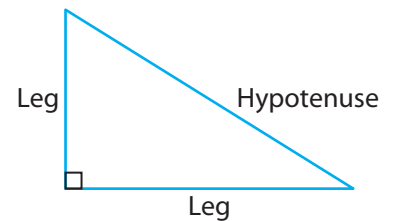
How can you prove the Pythagorean Theorem and use it to solve problems?

EXPLORE ACTIVITY

 **FL** 8.G.2.6

Proving the Pythagorean Theorem

In a right triangle, the two sides that form the right angle are the **legs**. The side opposite the right angle is the **hypotenuse**.

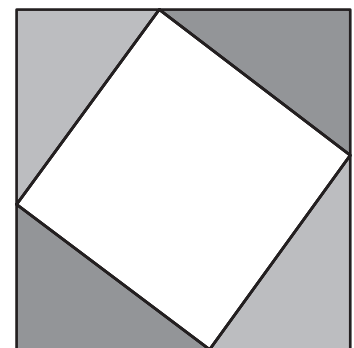


The Pythagorean Theorem

In a right triangle, the sum of the squares of the lengths of the legs is equal to the square of the length of the hypotenuse.

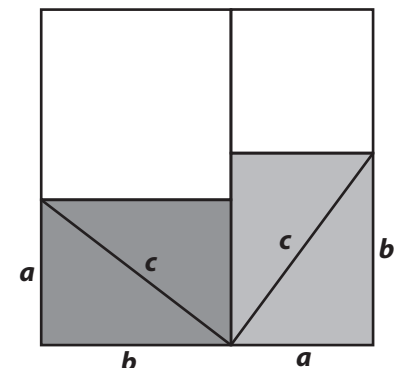
If a and b are legs and c is the hypotenuse, $a^2 + b^2 = c^2$.

- A** Draw a right triangle on a piece of paper and cut it out. Make one leg shorter than the other.
- B** Trace your triangle onto another piece of paper four times, arranging them as shown. For each triangle, label the shorter leg a , the longer leg b , and the hypotenuse c .
- C** What is the area of the unshaded square?



Label the unshaded square with its area.

- D** Trace your original triangle onto a piece of paper four times again, arranging them as shown. Draw a line outlining a larger square that is the same size as the figure you made in **B**.
- E** What is the area of the unshaded square at the top right of the figure in **D**? at the top left?



Label the unshaded squares with their areas.

- F** What is the total area of the unshaded regions in **D**?

Reflect

1. Explain whether the figures in **B** and **D** have the same area.

2. Explain whether the unshaded regions of the figures in **B** and **D** have the same area.

3. **Analyze Relationships** Write an equation relating the area of the unshaded region in step **B** to the unshaded region in **D**.

Using the Pythagorean Theorem

You can use the Pythagorean Theorem to find the length of a side of a right triangle when you know the lengths of the other two sides.



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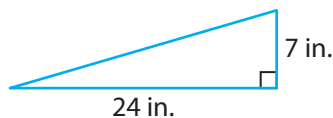
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EXAMPLE 1

FL 8.G.2.7

Find the length of the missing side.

A



$$a^2 + b^2 = c^2$$

$$24^2 + 7^2 = c^2$$

Substitute into the formula.

$$576 + 49 = c^2$$

Simplify.

$$625 = c^2$$

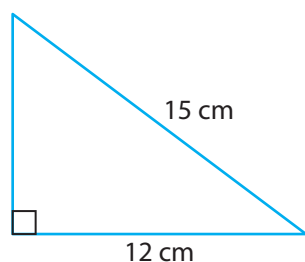
Add.

$$25 = c$$

Take the square root of both sides.

The length of the hypotenuse is 25 inches.

B



$$a^2 + b^2 = c^2$$

$$a^2 + 12^2 = 15^2$$

Substitute into the formula.

$$a^2 + 144 = 225$$

Simplify.

$$a^2 = 81$$

Use properties of equality to get a^2 by itself.

$$a = 9$$

Take the square root of both sides.

The length of the leg is 9 centimeters.

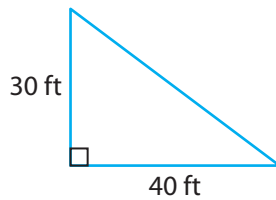
Math Talk

Mathematical Practices
If you are given the length of the hypotenuse and one leg, does it matter whether you solve for a or b ? Explain.

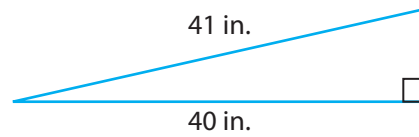
YOUR TURN

Find the length of the missing side.

4.



5.



Pythagorean Theorem in Three Dimensions

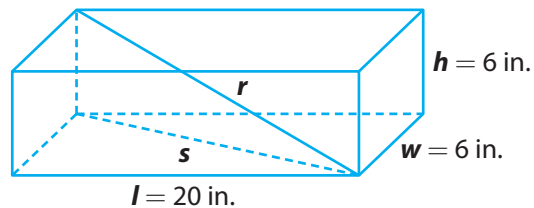
You can use the Pythagorean Theorem to solve problems in three dimensions.

EXAMPLE 2



FL 8.G.2.7

A box used for shipping narrow copper tubes measures 6 inches by 6 inches by 20 inches. What is the length of the longest tube that will fit in the box, given that the length of the tube must be a whole number of inches?



STEP 1

You want to find r , the length from a bottom corner to the opposite top corner. First, find s , the length of the diagonal across the bottom of the box.

$$w^2 + l^2 = s^2$$

$$6^2 + 20^2 = s^2 \quad \text{Substitute into the formula.}$$

$$36 + 400 = s^2 \quad \text{Simplify.}$$

$$436 = s^2 \quad \text{Add.}$$

STEP 2

Use your expression for s to find r .

$$h^2 + s^2 = r^2$$

$$6^2 + 436 = r^2 \quad \text{Substitute into the formula.}$$

$$472 = r^2 \quad \text{Add.}$$

$$\sqrt{472} = r \quad \text{Take the square root of both sides.}$$

$$21.7 \approx r \quad \text{Use a calculator to round to the nearest tenth.}$$

The length of the longest tube that will fit in the box is 21 inches.



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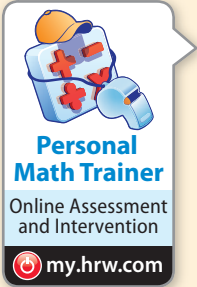
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Math Talk

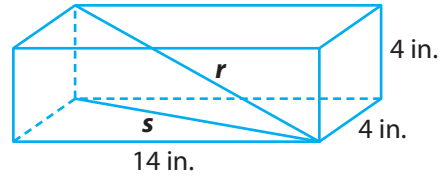
Mathematical Practices

Looking at Step 2, why did the calculations in Step 1 stop before taking the square root of both sides of the final equation?



YOUR TURN

6. Tina ordered a replacement part for her desk. It was shipped in a box that measures 4 in. by 4 in. by 14 in. What is the greatest length in whole inches that the part could have been?

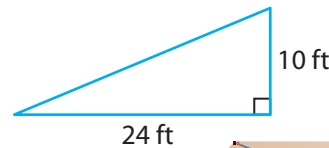


Guided Practice

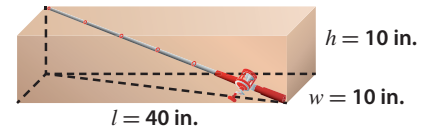
1. Find the length of the missing side of the triangle. (Explore Activity 1 and Example 1)

$$a^2 + b^2 = c^2 \rightarrow 24^2 + \square = c^2 \rightarrow \square = c^2$$

The length of the hypotenuse is feet.



2. Mr. Woo wants to ship a fishing rod that is 42 inches long to his son. He has a box with the dimensions shown. (Example 2)



- Find the square of the length of the diagonal across the bottom of the box.
- Find the length from a bottom corner to the opposite top corner to the nearest tenth. Will the fishing rod fit?



ESSENTIAL QUESTION CHECK-IN

3. State the Pythagorean Theorem and tell how you can use it to solve problems.

12.1 Independent Practice



FL 8.G.2.6, 8.G.2.7

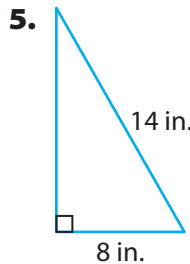
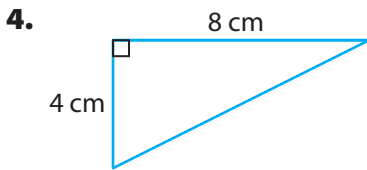


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Find the length of the missing side of each triangle. Round your answers to the nearest tenth.

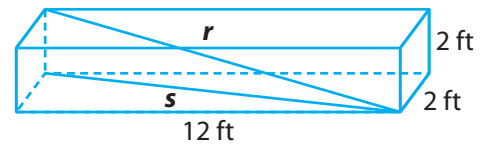


6. The diagonal of a rectangular big-screen TV screen measures 152 cm. The length measures 132 cm. What is the height of the screen? _____

7. Dylan has a square piece of metal that measures 10 inches on each side. He cuts the metal along the diagonal, forming two right triangles. What is the length of the hypotenuse of each right triangle to the nearest tenth of an inch? _____

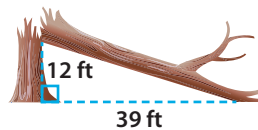
8. **Represent Real-World Problems** A painter has a 24-foot ladder that he is using to paint a house. For safety reasons, the ladder must be placed at least 8 feet from the base of the side of the house. To the nearest tenth of a foot, how high can the ladder safely reach? _____

9. What is the longest flagpole (in whole feet) that could be shipped in a box that measures 2 ft by 2 ft by 12 ft? _____



10. **Sports** American football fields measure 100 yards long between the end zones, and are $53\frac{1}{3}$ yards wide. Is the length of the diagonal across this field more or less than 120 yards? Explain.

11. **Justify Reasoning** A tree struck by lightning broke at a point 12 ft above the ground as shown. What was the height of the tree to the nearest tenth of a foot? Explain your reasoning.

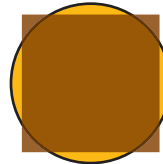




12. Multistep Main Street and Washington Avenue meet at a right angle. A large park begins at this corner. Joe's school lies at the opposite corner of the park. Usually Joe walks 1.2 miles along Main Street and then 0.9 miles up Washington Avenue to get to school. Today he walked in a straight path across the park and returned home along the same path. What is the difference in distance between the two round trips? Explain.

13. Analyze Relationships An isosceles right triangle is a right triangle with congruent legs. If the length of each leg is represented by x , what algebraic expression can be used to represent the length of the hypotenuse? Explain your reasoning.

14. Persevere in Problem Solving A square hamburger is centered on a circular bun. Both the bun and the burger have an area of 16 square inches.



a. How far, to the nearest hundredth of an inch, does each corner of the burger stick out from the bun? Explain.

b. How far does each bun stick out from the center of each side of the burger?

c. Are the distances in part **a** and part **b** equal? If not, which sticks out more, the burger or the bun? Explain.

LESSON 12.2 Converse of the Pythagorean Theorem

 **FL** 8.G.2.6

Explain a proof of the Pythagorean Theorem and its converse.



ESSENTIAL QUESTION

How can you test the converse of the Pythagorean Theorem and use it to solve problems?

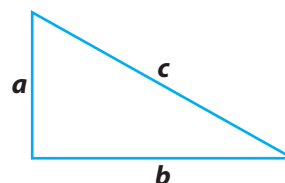
EXPLORE ACTIVITY

 **FL** 8.G.2.6

Testing the Converse of the Pythagorean Theorem

The Pythagorean Theorem states that if a triangle is a right triangle, then $a^2 + b^2 = c^2$.

The *converse* of the Pythagorean Theorem states that if $a^2 + b^2 = c^2$, then the triangle is a right triangle.

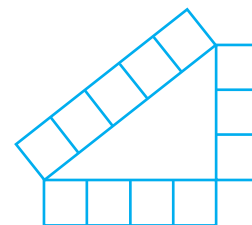


Decide whether the converse of the Pythagorean Theorem is true.

- A** Verify that the following sets of lengths make the equation $a^2 + b^2 = c^2$ true. Record your results in the table.

a	b	c	Is $a^2 + b^2 = c^2$ true?	Makes a right triangle?
3	4	5		
5	12	13		
7	24	25		
8	15	17		
20	21	29		

- B** For each set of lengths in the table, cut strips of grid paper with a width of one square and lengths that correspond to the values of a , b , and c .
- C** For each set of lengths, use the strips of grid paper to try to form a right triangle. An example using the first set of lengths is shown. Record your findings in the table.



Reflect

1. **Draw Conclusions** Based on your observations, explain whether you think the converse of the Pythagorean Theorem is true.



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Identifying a Right Triangle

The converse of the Pythagorean Theorem gives you a way to tell if a triangle is a right triangle when you know the side lengths.

EXAMPLE 1



FL 8.G.2.6

Tell whether each triangle with the given side lengths is a right triangle.

- A** 9 inches, 40 inches, and 41 inches

Let $a = 9$, $b = 40$, and $c = 41$.

$$a^2 + b^2 = c^2$$

$$9^2 + 40^2 \stackrel{?}{=} 41^2 \quad \text{Substitute into the formula.}$$

$$81 + 1600 \stackrel{?}{=} 1681 \quad \text{Simplify.}$$

$$1681 = 1681 \quad \text{Add.}$$

Since $9^2 + 40^2 = 41^2$, the triangle is a right triangle by the converse of the Pythagorean Theorem.

- B** 8 meters, 10 meters, and 12 meters

Let $a = 8$, $b = 10$, and $c = 12$.

$$a^2 + b^2 = c^2$$

$$8^2 + 10^2 \stackrel{?}{=} 12^2 \quad \text{Substitute into the formula.}$$

$$64 + 100 \stackrel{?}{=} 144 \quad \text{Simplify.}$$

$$164 \neq 144 \quad \text{Add.}$$

Since $8^2 + 10^2 \neq 12^2$, the triangle is not a right triangle by the converse of the Pythagorean Theorem.

YOUR TURN

Tell whether each triangle with the given side lengths is a right triangle.

2. 14 cm, 23 cm, and 25 cm

3. 16 in., 30 in., and 34 in.

4. 27 ft, 36 ft, 45 ft

5. 11 mm, 18 mm, 21 mm



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Using the Converse of the Pythagorean Theorem

You can use the converse of the Pythagorean Theorem to solve real-world problems.



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EXAMPLE 2



FL 8.G.2.6

Katya is buying edging for a triangular flower garden she plans to build in her backyard. If the lengths of the three pieces of edging that she purchases are 13 feet, 10 feet, and 7 feet, will the flower garden be in the shape of a right triangle?

Use the converse of the Pythagorean Theorem. Remember to use the longest length for c .

Let $a = 7$, $b = 10$, and $c = 13$.

$$a^2 + b^2 = c^2$$

$$7^2 + 10^2 \stackrel{?}{=} 13^2 \quad \text{Substitute into the formula.}$$

$$49 + 100 \stackrel{?}{=} 169 \quad \text{Simplify.}$$

$$149 \neq 169 \quad \text{Add.}$$

Since $7^2 + 10^2 \neq 13^2$, the garden will not be in the shape of a right triangle.

Math Talk

Mathematical Practices



To what length, to the nearest tenth, can Katya trim the longest piece of edging to form a right triangle?

YOUR TURN

6. A blueprint for a new triangular playground shows that the sides measure 480 ft, 140 ft, and 500 ft. Is the playground in the shape of a right triangle? Explain.

7. A triangular piece of glass has sides that measure 18 in., 19 in., and 25 in. Is the piece of glass in the shape of a right triangle? Explain.

8. A corner of a fenced yard forms a right angle. Can you place a 12 foot long board across the corner to form a right triangle for which the leg lengths are whole numbers? Explain.



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Guided Practice

1. Lashandra used grid paper to construct the triangle shown. (Explore Activity)

a. What are the lengths of the sides of Lashandra's triangle?

_____ units, _____ units, _____ units

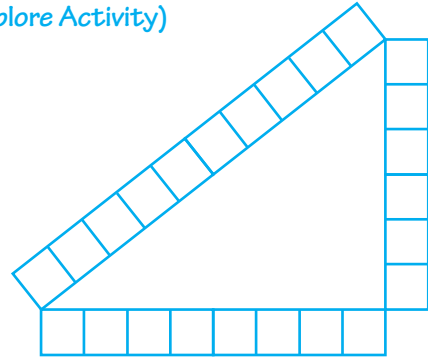
b. Use the converse of the Pythagorean Theorem to determine whether the triangle is a right triangle.

$$a^2 + b^2 = c^2$$

$$\square^2 + \square^2 \stackrel{?}{=} \square^2$$

$$\square + \square \stackrel{?}{=} \square$$

$$\square \stackrel{?}{=} \square$$



The triangle that Lashandra constructed **is / is not** a right triangle.

2. A triangle has side lengths 9 cm, 12 cm, and 16 cm. Tell whether the triangle is a right triangle. (Example 1)

Let $a =$ _____, $b =$ _____, and $c =$ _____.

$$a^2 + b^2 = c^2$$

$$\square^2 + \square^2 \stackrel{?}{=} \square^2$$

$$\square + \square \stackrel{?}{=} \square$$

$$\square \stackrel{?}{=} \square$$

By the converse of the Pythagorean Theorem, the triangle **is / is not** a right triangle.

3. The marketing team at a new electronics company is designing a logo that contains a circle and a triangle. On one design, the triangle's side lengths are 2.5 in., 6 in., and 6.5 in. Is the triangle a right triangle? Explain. (Example 2)




ESSENTIAL QUESTION CHECK-IN

4. How can you use the converse of the Pythagorean Theorem to tell if a triangle is a right triangle?

12.2 Independent Practice

 **FL** 8.G.2.6



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Tell whether each triangle with the given side lengths is a right triangle.

5. 11 cm, 60 cm, 61 cm

6. 5 ft, 12 ft, 15 ft

7. 9 in., 15 in., 17 in.

8. 15 m, 36 m, 39 m

9. 20 mm, 30 mm, 40 mm

10. 20 cm, 48 cm, 52 cm

11. 18.5 ft, 6 ft, 17.5 ft

12. 2 mi, 1.5 mi, 2.5 mi

13. 35 in., 45 in., 55 in.

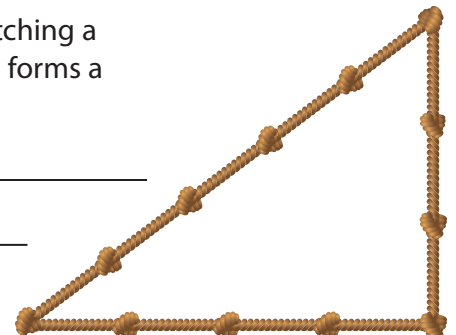
14. 25 cm, 14 cm, 23 cm

15. The emblem on a college banner consists of the face of a tiger inside a triangle. The lengths of the sides of the triangle are 13 cm, 14 cm, and 15 cm. Is the triangle a right triangle? Explain.

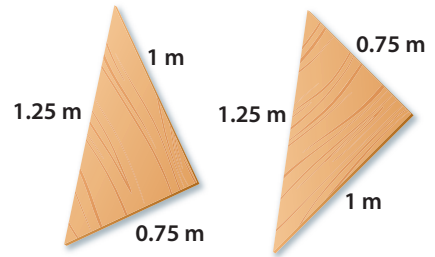
16. Kerry has a large triangular piece of fabric that she wants to attach to the ceiling in her bedroom. The sides of the piece of fabric measure 4.8 ft, 6.4 ft, and 8 ft. Is the fabric in the shape of a right triangle? Explain.

17. A mosaic consists of triangular tiles. The smallest tiles have side lengths 6 cm, 10 cm, and 12 cm. Are these tiles in the shape of right triangles? Explain.

18. History In ancient Egypt, surveyors made right angles by stretching a rope with evenly spaced knots as shown. Explain why the rope forms a right angle.



19. **Justify Reasoning** Yoshi has two identical triangular boards as shown. Can he use these two boards to form a rectangle? Explain.



20. **Critique Reasoning** Shoshanna says that a triangle with side lengths 17 m, 8 m, and 15 m is not a right triangle because $17^2 + 8^2 = 353$, $15^2 = 225$, and $353 \neq 225$. Is she correct? Explain.

H.O.T. FOCUS ON HIGHER ORDER THINKING

21. **Make a Conjecture** Diondre says that he can take any right triangle and make a new right triangle just by doubling the side lengths. Is Diondre's conjecture true? Test his conjecture using three different right triangles.

22. **Draw Conclusions** A diagonal of a parallelogram measures 37 inches. The sides measure 35 inches and 1 foot. Is the parallelogram a rectangle? Explain your reasoning.

23. **Represent Real-World Problems** A soccer coach is marking the lines for a soccer field on a large recreation field. The dimensions of the field are to be 90 yards by 48 yards. Describe a procedure she could use to confirm that the sides of the field meet at right angles.

Work Area

LESSON 12.3 Distance Between Two Points

 **FL** 8.G.2.8

Apply the Pythagorean Theorem to find the distance between two points in a coordinate system.



ESSENTIAL QUESTION

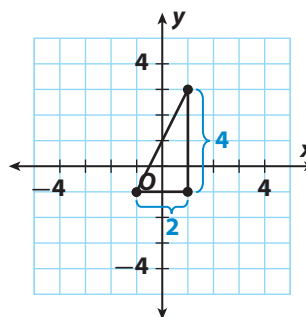
How can you use the Pythagorean Theorem to find the distance between two points on a coordinate plane?

Pythagorean Theorem in the Coordinate Plane

EXAMPLE 1

 **FL** 8.G.2.8

The figure shows a right triangle. Approximate the length of the hypotenuse to the nearest tenth using a calculator.



STEP 1 Find the length of each leg.

The length of the vertical leg is 4 units.

The length of the horizontal leg is 2 units.

STEP 2 Let $a = 4$ and $b = 2$. Let c represent the length of the hypotenuse. Use the Pythagorean Theorem to find c .

$$a^2 + b^2 = c^2$$

$$4^2 + 2^2 = c^2 \quad \text{Substitute into the formula.}$$

$$20 = c^2 \quad \text{Add.}$$

$$\sqrt{20} = c \quad \text{Take the square root of both sides.}$$

$$\sqrt{20} \approx 4.5 \quad \text{Use a calculator and round to the nearest tenth.}$$

STEP 3 Check for reasonableness by finding perfect squares close to 20.

$$\sqrt{20} \text{ is between } \sqrt{16} \text{ and } \sqrt{25}, \text{ or } \sqrt{16} < \sqrt{20} < \sqrt{25}.$$

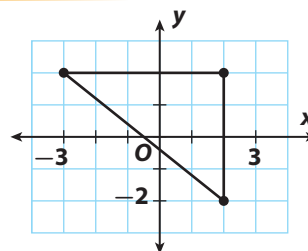
$$\text{Simplifying gives } 4 < \sqrt{20} < 5.$$

Since 4.5 is between 4 and 5, the answer is reasonable.

The hypotenuse is about 4.5 units long.

YOUR TURN

1. Approximate the length of the hypotenuse to the nearest tenth using a calculator.



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Finding the Distance Between Any Two Points

The Pythagorean Theorem can be used to find the distance between any two points (x_1, y_1) and (x_2, y_2) in the coordinate plane. The resulting expression is called the Distance Formula.

Distance Formula

In a coordinate plane, the distance d between two points (x_1, y_1) and (x_2, y_2) is

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Use the Pythagorean Theorem to derive the Distance Formula.

A To find the distance between points P and Q , draw segment \overline{PQ} and label its length d . Then draw horizontal segment \overline{PR} and vertical segment \overline{QR} . Label the lengths of these segments a and b . Triangle PQR is a _____ triangle, with hypotenuse _____.

B Since \overline{PR} is a horizontal segment, its length, a , is the difference between its x -coordinates. Therefore, $a = x_2 -$ _____.

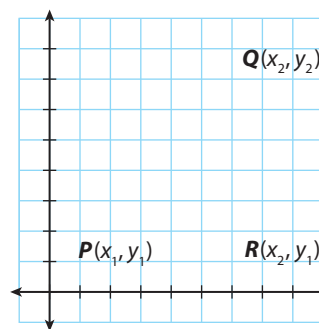
C Since \overline{QR} is a vertical segment, its length, b , is the difference between its y -coordinates. Therefore, $b = y_2 -$ _____.

D Use the Pythagorean Theorem to find d , the length of segment \overline{PQ} . Substitute the expressions from **B** and **C** for a and b .

$$d^2 = a^2 + b^2$$

$$d = \sqrt{a^2 + b^2}$$

$$d = \sqrt{\left(\square - \square\right)^2 + \left(\square - \square\right)^2}$$



Math Talk
Mathematical Practices
 What do $x_2 - x_1$ and $y_2 - y_1$ represent in terms of the Pythagorean Theorem?

Reflect

2. Why are the coordinates of point R the ordered pair (x_2, y_1) ?

Finding the Distance Between Two Points

The Pythagorean Theorem can be used to find the distance between two points in a real-world situation. You can do this by using a coordinate grid that overlays a diagram of the real-world situation.



Math On the Spot

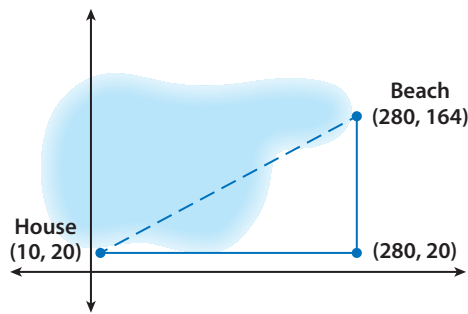
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EXAMPLE 2



FL 8.G.2.8

Francesca wants to find the distance between her house on one side of a lake and the beach on the other side. She marks off a third point forming a right triangle, as shown. The distances in the diagram are measured in meters.



Use the Pythagorean Theorem to find the straight-line distance from Francesca's house to the beach.

STEP 1 Find the length of the horizontal leg.

The length of the horizontal leg is the absolute value of the difference between the x-coordinates of the points (280, 20) and (10, 20).

$$|280 - 10| = 270$$

The length of the horizontal leg is 270 meters.

STEP 2 Find the length of the vertical leg.

The length of the vertical leg is the absolute value of the difference between the y-coordinates of the points (280, 164) and (280, 20).

$$|164 - 20| = 144$$

The length of the vertical leg is 144 meters.

STEP 3 Let $a = 270$ and $b = 144$. Let c represent the length of the hypotenuse. Use the Pythagorean Theorem to find c .

$$a^2 + b^2 = c^2$$

$$270^2 + 144^2 = c^2 \quad \text{Substitute into the formula.}$$

$$72,900 + 20,736 = c^2 \quad \text{Simplify.}$$

$$93,636 = c^2 \quad \text{Add.}$$

$$\sqrt{93,636} = c \quad \text{Take the square root of both sides.}$$

$$306 = c \quad \text{Simplify.}$$

The distance from Francesca's house to the beach is 306 meters.

Math Talk

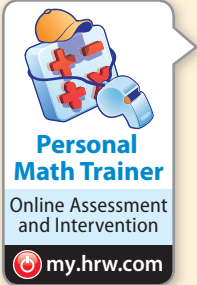


Mathematical Practices

Why is it necessary to take the absolute value of the coordinates when finding the length of a segment?

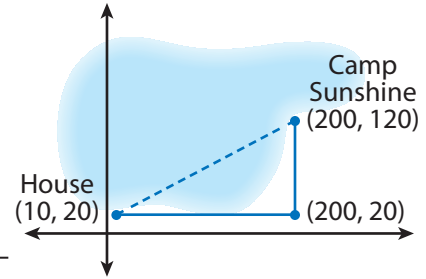
Reflect

3. Show how you could use the Distance Formula to find the distance from Francesca's house to the beach.



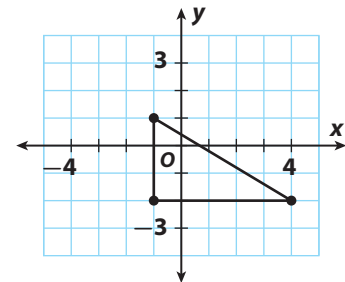
YOUR TURN

4. Camp Sunshine is also on the lake. Use the Pythagorean Theorem to find the distance between Francesca's house and Camp Sunshine to the nearest tenth of a meter.

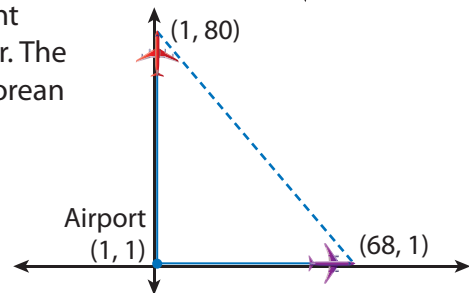


Guided Practice

- Approximate the length of the hypotenuse of the right triangle to the nearest tenth using a calculator. (Example 1) _____
- Find the distance between the points $(3, 7)$ and $(15, 12)$ on the coordinate plane. (Explore Activity) _____
- A plane leaves an airport and flies due north. Two minutes later, a second plane leaves the same airport flying due east. The flight plan shows the coordinates of the two planes 10 minutes later. The distances in the graph are measured in miles. Use the Pythagorean Theorem to find the distance shown between the two planes.



(Example 2) _____




ESSENTIAL QUESTION CHECK-IN

4. Describe two ways to find the distance between two points on a coordinate plane.


12.3 Independent Practice

 **FL** 8.G.2.8

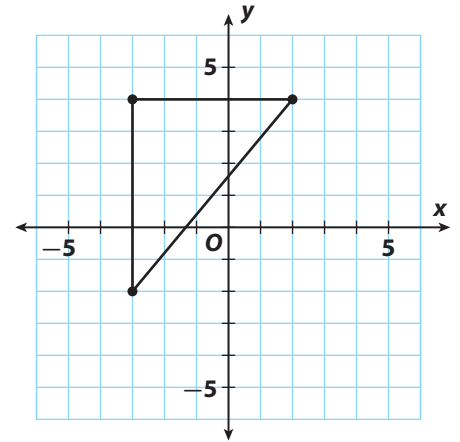


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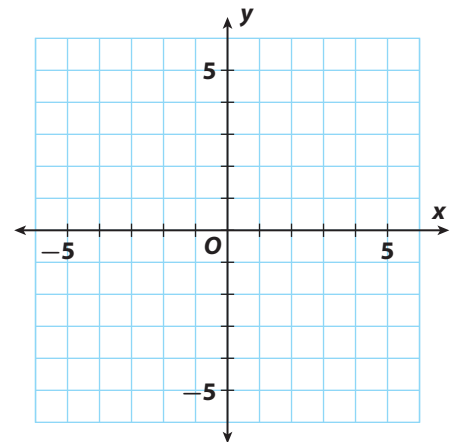


5. A metal worker traced a triangular piece of sheet metal on a coordinate plane, as shown. The units represent inches. What is the length of the longest side of the metal triangle? Approximate the length to the nearest tenth of an inch using a calculator. Check that your answer is reasonable.



6. When a coordinate grid is superimposed on a map of Harrisburg, the high school is located at (17, 21) and the town park is located at (28, 13). If each unit represents 1 mile, how many miles apart are the high school and the town park? Round your answer to the nearest tenth.

7. The coordinates of the vertices of a rectangle are given by $R(-3, -4)$, $E(-3, 4)$, $C(4, 4)$, and $T(4, -4)$. Plot these points on the coordinate plane at the right and connect them to draw the rectangle. Then connect points E and T to form diagonal \overline{ET} .

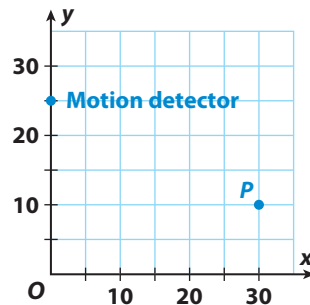


- a. Use the Pythagorean Theorem to find the exact length of \overline{ET} .
- _____
- b. How can you use the Distance Formula to find the length of \overline{ET} ? Show that the Distance Formula gives the same answer.

8. **Multistep** The locations of three ships are represented on a coordinate grid by the following points: $P(-2, 5)$, $Q(-7, -5)$, and $R(2, -3)$. Which ships are farthest apart?

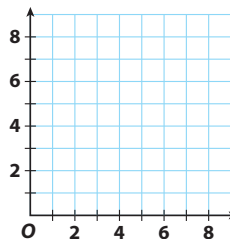
9. **Make a Conjecture** Find as many points as you can that are 5 units from the origin. Make a conjecture about the shape formed if all the points 5 units from the origin were connected.

10. **Justify Reasoning** The graph shows the location of a motion detector that has a maximum range of 34 feet. A peacock at point P displays its tail feathers. Will the motion detector sense this motion? Explain.



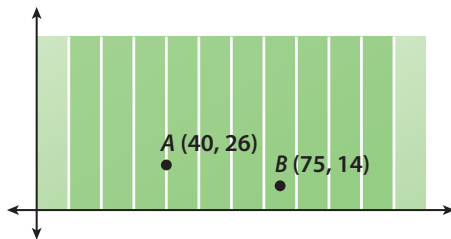
H.O.T. FOCUS ON HIGHER ORDER THINKING

11. **Persevere in Problem Solving** One leg of an isosceles right triangle has endpoints $(1, 1)$ and $(6, 1)$. The other leg passes through the point $(6, 2)$. Draw the triangle on the coordinate plane. Then show how you can use the Distance Formula to find the length of the hypotenuse. Round your answer to the nearest tenth.



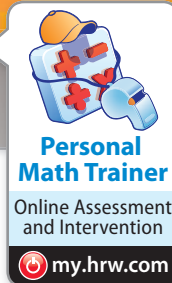
12. **Represent Real-World Problems**

The figure shows a representation of a football field. The units represent yards. A sports analyst marks the locations of the football from where it was thrown (point A) and where it was caught (point B). Explain how you can use the Pythagorean Theorem to find the distance the ball was thrown. Then find the distance.



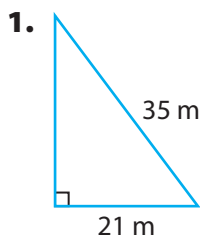
Work Area

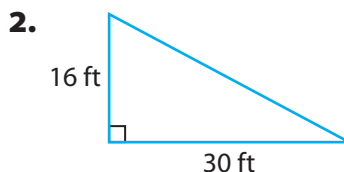
Ready to Go On?



12.1 The Pythagorean Theorem

Find the length of the missing side.





12.2 Converse of the Pythagorean Theorem

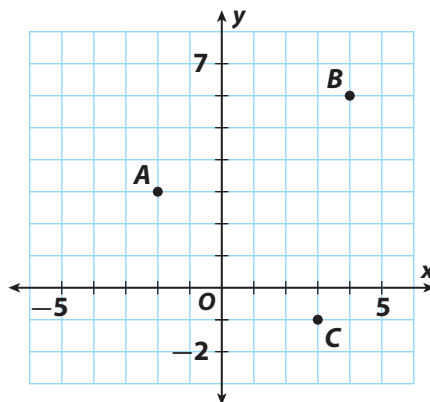
Tell whether each triangle with the given side lengths is a right triangle.

- 3. 11, 60, 61 _____
- 4. 9, 37, 40 _____
- 5. 15, 35, 38 _____
- 6. 28, 45, 53 _____
- 7. Keelie has a triangular-shaped card. The lengths of its sides are 4.5 cm, 6 cm, and 7.5 cm. Is the card a right triangle? _____

12.3 Distance Between Two Points

Find the distance between the given points. Round to the nearest tenth.

- 8. A and B _____
- 9. B and C _____
- 10. A and C _____



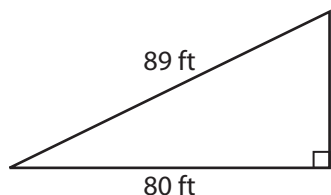
ESSENTIAL QUESTION

- 11. How can you use the Pythagorean Theorem to solve real-world problems?

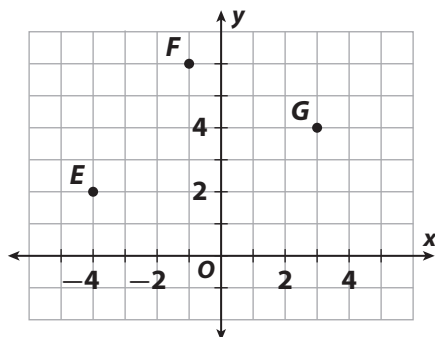


Selected Response

1. What is the missing length of the side?



- (A) 9 ft (C) 39 ft
(B) 30 ft (D) 120 ft
2. Which relation does **not** represent a function?
- (A) (0, 8), (3, 8), (1, 6)
(B) (4, 2), (6, 1), (8, 9)
(C) (1, 20), (2, 23), (9, 26)
(D) (0, 3), (2, 3), (2, 0)
3. Two sides of a right triangle have lengths of 72 cm and 97 cm. The third side is **not** the hypotenuse. How long is the third side?
- (A) 25 cm (C) 65 cm
(B) 45 cm (D) 121 cm
4. To the nearest tenth, what is the distance between point *F* and point *G*?



- (A) 4.5 units (C) 7.3 units
(B) 5.0 units (D) 20 units

5. A flagpole is 53 feet tall. A rope is tied to the top of the flagpole and secured to the ground 28 feet from the base of the flagpole. What is the length of the rope?

- (A) 25 feet (C) 53 feet
(B) 45 feet (D) 60 feet

6. Which set of lengths are **not** the side lengths of a right triangle?

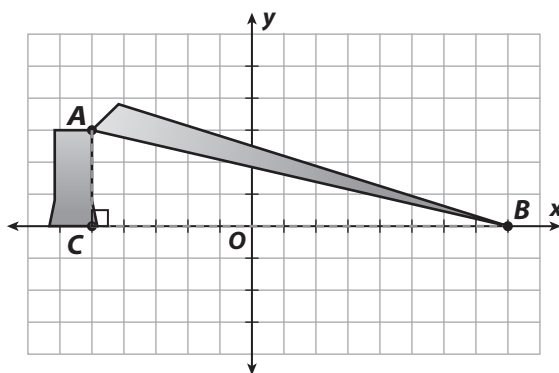
- (A) 36, 77, 85 (C) 27, 120, 123
(B) 20, 99, 101 (D) 24, 33, 42

7. A triangle has one right angle. What could the measures of the other two angles be?

- (A) 25° and 65° (C) 55° and 125°
(B) 30° and 15° (D) 90° and 100°

Mini-Task

8. A fallen tree is shown on the coordinate grid below. Each unit represents 1 meter.



- a. What is the distance from *A* to *B*?

- b. What was the height of the tree before it fell?
